

## Mathematical Models In The Assessment Of Infective Force In Filariasis

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Conventional indices in use for assessment of endemicity of filariasis are infection rate, endemicity rate, average infestation and, more recently, median microfilaria density. Going a step ahead, the present paper indicates how 'catalytic models' can be gainfully utilized to measure, in concrete and specific terms, the infective force of filariasis acting on a population, for which distribution of proportion positive cases by age is known. One of the two models discussed in the paper estimates the reversion force also acting on positive cases. Epidemiological utility of such model lies in its predictive value for projecting proportion positive cases at certain point of time given the infective and reversion forces, or vice-versa, and in comparing the endemicity levels of two or more areas as also at different times, the latter of which may indicate efficiency of the control measures operating in an area.

### Introduction

Epidemiological assessment of filariasis in a community poses many complex problems. Many important links in the natural history of filariasis are still missing. Various indices used for the assessment of filariasis include infection (microfilaria positive) rate, disease rate, endemicity rate and average infestation, i.e. the average number of microfilaria per positive night blood smear in the population surveyed. More recently (Sasa 1967, Srivastava et al 1969), distribution of microfilaria counts has been used to compute median microfilaria density ( $MfD_{50}$ ) and to draw inferences therefrom.

The objective of the present paper is to go one step ahead and to measure the infective force acting on the population producing microfilaria positive cases. Two types of models have been fitted to a filaria survey data (Srivastava and Prasad 1969). An attempt to bring out the relative efficiency of the two models in this particular case has also been made, suggesting which of the two models describe the situation more adequately. The measure of infective force may be used to compare the endemicity of filariasis in two areas or at two times including effectiveness of filaria control measures.

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### Material and Methods

If an 'effective contact' is assumed to mean a contact sufficient to produce infection in a susceptible subject, then the force of infection can be measured in terms of effective contacts per unit of time per individual. Muench (1959) has aptly called it as a catalytic force acting on the population which, in our context, better be understood as annual rate of becoming positive for microfilaria. He has proposed catalytic models describing relationship between proportion positive population, time and infective force. The models hold good under certain conditions (Muench 1959) which are none too rigorous in view of the simplicity introduced by and the utility of such relationships.

According to simple catalytic model,

$$y = 1 - e^{-rt},$$

where  $y$  is the proportion positive population at time  $t$  and  $r$  is the infective force per unit of time per individual.  $y$  has been assumed zero at time zero.

On the hypothesis that a reverting force, in the form of some reaction, is also active along with the infective force, the catalytic model, reversible type, takes the form (Muench 1959)

$$y = \frac{a}{a+b} \left[ 1 - e^{-(a+b)t} \right],$$

where 'a' is the infective force producing positive cases, while 'b' is the force reverting positive cases to negative ones, and again,  $y=0$  at  $t=0$ . Both these models have been fitted to the distribution of filaria positive cases by age obtained from data of filaria survey same villages of the Rural Health Training Centre, Sarojini Nagar, Lucknow. The technique of fitting and notation used are the same as advocated by Muench (1959), involving estimation of  $r$ , which is  $(a+b)$  in the latter model, by the method of moments, approximated by the chart given by him. Efficiency of the two models in representing the data, which may also be the accuracy of the infective force resulting from the models, have been judged by closeness of fit of the respective models to the same data.

### Results

**Estimation of infective force :** Calculations needed for fitting simple catalytic model are shown in Table I. The infective force in this case is estimated as  $r=0.005$ , showing that, on average, 5 individuals were becoming positive to filaria per year per 1000 susceptibles. Expected number of positive cases in respective age-groups on the basis of the model is given in column 9 of Table I. The observed graph and the expected curve based on cols. 5 and 8 of Table I are shown in the Graph.

Introducing now a force responsible for losing positivity and fitting reversible catalytic model, the estimates of parameters are

$$a=0.014 \text{ and } b=0.061$$

as per details shown in Table II. This gives an enhanced rate of being positive at 14 per year per 1000 susceptible individuals.

At the same time, a reverting force estimated to be active at the rate of 61 cases losing positivity per year per 1000 positive cases was also working. This fit, given by column 6 of Table II, is also shown in the Graph. The latter fit is relatively more close to the observed data as is apparent from the Graph also.

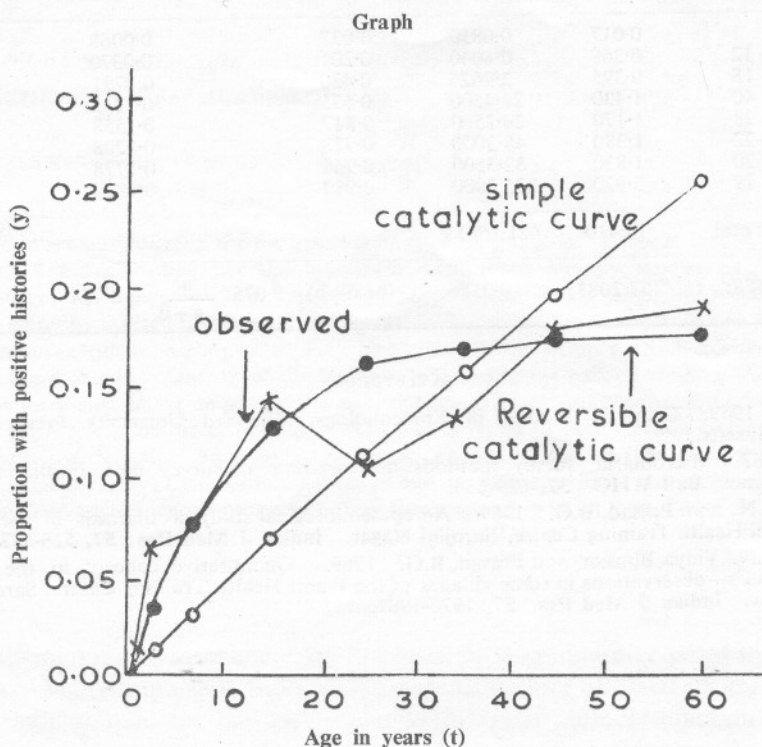


Table I. Calculation of infective force by simple catalytic model

Age group (years)	Mid-point (t)	Number examined (n)	Cases with +ve histories (0)	Proportion with +ve histories (y)	Width of age interval (w)	$y \times w$ (A)	$y_1 = 1 - e^{-rt}$	$n \cdot y_1$ ( $E_1$ )
0-1	0.5	58	1	0.017	1	0.017	0.0025	0
1-5	3.0	179	12	0.067	4	0.268	0.0150	3
5-10	7.5	227	18	0.079	5	0.395	0.0370	8
10-20	15.0	278	40	0.143	10	1.430	0.0720	20
20-30	25.0	239	28	0.117	10	1.170	0.1180	28
30-40	35.0	159	22	0.138	10	1.380	0.1600	27
40-50	45.0	109	20	0.183	10	1.830	0.2010	22
50-70	60.0	97	19	0.196	20	3.920	0.2590	25

$$\Sigma A = 10.410; \Sigma E_1 = 14.87; r' = 0.0035; r = 0.005$$

Table II. Calculation of infective and reversion forces by reversible catalytic model

Mid-point (t)	Observed +ve cases (0)	A (col. 7 of Table I)	tA	$1 - e^{-(a+b)t}$	$y_t = \frac{a}{a+b} [1 - e^{-(a+b)t}] E_t = ny_t$	
0.5	1	0.017	0.0850	0.037	0.0068	0
3.0	12	0.268	0.8040	0.201	0.0370	7
7.5	18	0.395	2.9625	0.430	0.0791	18
15.0	40	1.430	21.4500	0.675	0.1242	35
25.0	28	1.170	29.2500	0.847	0.1558	36
35.0	22	1.380	48.3000	0.928	0.1708	27
45.0	20	1.830	82.3500	0.966	0.1778	19
60.0	19	3.920	235.2000	0.989	0.1820	18
Total		10.410	421.1665			
$\bar{t} = 40.4578$ ; $t^{-1} = 57.2083$ ; $r' = 0.0526$ ; $r' \text{ (or } a+b) = 0.075$ ; $\frac{a}{a+b} = 0.184$						

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